# Infrared and Ultraviolet Coupling in QCD

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# INFRARED AND ULTRAVIOLET COUPLING IN QCD

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The coupled Dyson-Schwinger equations for the gluon and ghost propagators in QCD are shown to have solutions that correspond to a unique running coupling that has a finite infrared fixed point and the expected logarithmic decrease in the ultraviolet. The infrared coupling is large enough to support chiral symmetry breaking; and quarks are not confined, but they cannot be isolated.

## 1 Introduction

A brief — and biased — history of the subject might start with Mandelstam's 1979 work on Dyson-Schwinger (DS) equations in QCD [1]: in this paper the possibility of an infrared singular gluon propagator was introduced, and the logarithms in the ultraviolet behaviour of solutions of the gluon equation were also investigated. In the same year, Blatt and I showed that the rainbow approximation to the fermion DS equation exhibits first Riemann sheet complex singularities [2]. The stage was set for detailed analyses of the gluon and the quark DS equations [3], with a view to investigating both confinement and chiral symmetry breaking. Ten years later, Pennington et al. succeeded in alleviating one worrying feature of the Mandelstam approximation by projecting out the most divergent term in the gluon equation [4].

A new possibility has been opened up by the papers of von Smekal, Hauck and Alkofer [5]. In this work the coupling of the gluon to the ghost was not neglected. These authors claim that it is not the gluon, but rather the ghost propagator that is highly singular in the infrared limit. The running coupling itself has a *finite* though quite large value in the limit of zero energy, presumably large enough to guarantee chiral symmetry breaking in the quark equation. Quarks would technically not be confined, in the sense that the energy needed to separate quarks and antiquarks from a bound state to spatial infinity would be finite. However, so long as it is energetically more favourable for stretched gluon strings to break, with creation of light quark-antiquark pairs, one might reasonably hope for an effective *de facto* confinement, in the sense that quarks, and other coloured states, could not be isolated.

In the bulk of this paper, as in the talk it recalls, I shall consider the DS equations for the gluon and ghost propagators, with the gluon-ghost vertex replaced by its bare value, and all other vertices turned off (i.e. the gluon-gluon vertices and the gluon-quark vertices are replaced by zero). Despite the grossness of this approximation, there is reason to believe that the correct infrared asymptotics is reproduced. In the ultraviolet, it is true, the gluon-gluon and gluon-quark interactions are numerically more significant than the gluon-ghost term, but even here the expected inverse logarithmic decrease of the running coupling is manifested by the solution of our mutilated equations — it is only the coefficient of the logarithm that is wrong. An advantage of the approximation is that the renormalization scalings of exact QCD are preserved: the running coupling that is calculated from the equations contains only one parameter, namely $\Lambda_{\rm QCD}$. In the last section we will indicate what has been done, and what remains to be accomplished, when the other vertices are also included. An open problem is precisely this question of the renormalization group invariance of approximated systems of DS equations.

## 2 Approximate Dyson-Schwinger Equations

We shall write the gluon propagator in Landau gauge as

$$D^{ab}_{\mu\nu}(p) = -\delta^{ab}\frac{1}{p^2}\Delta_{\mu\nu}(p)F(-p^2)\,,$$

where $a$ and $b$ are colour indices, and where $\Delta = \Delta^2$ is the projection operator $\Delta_{\mu\nu}(p) = g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}$. The ghost propagator will be written in the form

$$G^{ab}(p) = -\delta^{ab}\frac{1}{p^2}G(-p^2)\,,$$

and we shall refer to the scalar functions $F$ and $G$ as the gluon and ghost form factors, respectively.

The claim made by von Smekal et al. is that, in the limit $x = -p^2 \to 0$, these form factors have the following behaviours:

$$F(x) \sim x^{2\kappa} \qquad G(x) \sim x^{-\kappa}\,, \tag{1}$$

where $\kappa \approx 0.92$. To obtain these results certain Ansätze were made for the three-gluon and ghost-gluon vertices, functional forms inspired, but not uniquely determined by Slavnov-Taylor identities. In fact the Ansatz made for the ghost-gluon vertex is such that actually the infrared behaviour Eq.(1) is not consistent with the Dyson-Schwinger equations. The difficulty is the occurrence of a term

$$\int_x^{\Lambda^2} \frac{dy}{y} F(y)G^2(y) \tag{2}$$

in the equation for the ghost form factor, which, with the form (1), would yield an impermissible $\log x$ factor in the limit $x \to 0$. Von Smekal et al. circumvent this problem by replacing *one* of the factors $G(y)$ by $G(x)$, thereby undermining to a large extent their supposed improvement of the vertex Ansatz.

Since we found the *ad hoc* nature of this last replacement questionable, we decided first to see what would happen if one simply replaces the full vertices by bare ones. In this case the problematic logarithm of Eq.(2) does not occur, and we can simply analyze the equation as it stands. If the behaviour (1) were to go away, it would bode ill for the new approach. However, our finding is that, with bare vertices, the form (1) indeed remains good, but with the index changed to $\kappa \approx 0.77$. Moreover, we can show that the solutions of the coupled gluon and ghost equations lie on a three-dimensional manifold, i.e. the general solution has three free parameters; nevertheless all solutions have the infrared behaviour (1) and they all refer to the same, unique physical situation.

In the approximation that all vertices are set equal to zero except the gluon-ghost vertex — for which the bare vertex is taken — the equations for the renormalized gluon and ghost form factors in Landau gauge are as follows:

$$F^{-1}(p^2) = Z_3 + \frac{g^2}{4\pi^3}\tilde{Z}_1 \int_0^{\Lambda^2} \frac{dq^2}{p^2} G(q^2) \int_0^{\pi} d\theta \sin^2\theta\, M(p^2, q^2, r^2) G(r^2)$$

$$G^{-1}(p^2) = \tilde{Z}_3 - \frac{3g^2}{4\pi^3}\tilde{Z}_1 \int_0^{\Lambda^2} dq^2 q^2 G(q^2) \int_0^{\pi} d\theta \frac{\sin^4\theta}{r^4} F(r^2)\,,$$

where $g$ is the renormalized QCD coupling, where $r^2 = p^2 + q^2 - 2pq\cos\theta$ and

$$M(p^2, q^2, r^2) = \frac{1}{r^2}\left(\frac{p^2+q^2}{2} - \frac{q^4}{p^2}\right) + \frac{1}{2} + \frac{2q^2}{p^2} - \frac{r^2}{p^2}\,.$$

We shall further employ angular averaging, which consists in replacing $r^2$ in $F(r^2)$ and $G(r^2)$ by the maximum of $p^2$ and $q^2$. The coupled equations for $F$ and $G$ then take on the following form:

$$F^{-1}(x) = Z_3 + \lambda\left[G(x)\int_0^x \frac{dy}{x}\left(\frac{3y}{2x} - \frac{y^2}{x^2}\right) G(y) + \int_x^{\Lambda^2} \frac{dy}{2y} G^2(y)\right] \qquad (3)$$

and

$$G^{-1}(x) = \tilde{Z}_3 - \frac{9}{4}\lambda\left[\frac{F(x)}{x^2}\int_0^x dy y G(y) + \int_x^{\Lambda^2} \frac{dy}{y} F(y) G(y)\right], \qquad (4)$$

where $x = p^2$, $y = q^2$, and $\lambda = \frac{g^2}{16\pi^2}\tilde{Z}_1$.

## 3 Infrared Behaviour

Make a subtraction at $x = 1$ (say) and try the Ansatz

$$F(x) = Ax^{\alpha} \qquad G(x) = Bx^{\beta}\,, \tag{5}$$

for $x \to 0$. This gives

$$A^{-1}x^{-\alpha} = A^{-1} + \lambda B^2 \left[\frac{3}{2}\frac{1}{2+\beta} - \frac{1}{3+\beta} - \frac{1}{4\beta}\right](x^{2\beta} - 1) \tag{6}$$

and

$$B^{-1}x^{-\beta} = B^{-1} - \frac{9}{4}\lambda \left[\frac{1}{2+\beta} - \frac{1}{\alpha+\beta}\right](x^{\alpha+\beta} - 1) \tag{7}$$

on condition that $\beta > -2$. The powers on both sides of Eq.(6)-(7) agree if $\alpha = -2\beta$. Under this condition, both the constant and the power terms in Eq.(6) match if

$$A^{-1} = \lambda B^2 \left[\frac{3}{2}\frac{1}{2+\beta} - \frac{1}{3+\beta} - \frac{1}{4\beta}\right]\,,$$

while both terms in Eq.(7) match if

$$B^{-1} = -\frac{9}{4}\lambda AB \left[\frac{1}{2+\beta} - \frac{1}{\alpha+\beta}\right]\,.$$

Can these conditions be met simultaneously? Let us set $\alpha = 2\kappa$ and $\beta = -\kappa$. The above two equations can be written

$$\frac{1}{\lambda AB^2} = \frac{3}{2(2-\kappa)} - \frac{1}{3-\kappa} + \frac{1}{4\kappa} = \frac{9}{4}\left(\frac{1}{\kappa} - \frac{1}{2-\kappa}\right)\,. \tag{8}$$

This implies a restriction on the index $\kappa$, which remarkably does not depend on the value of the coupling strength, $\lambda$:

$$19\kappa^2 - 77\kappa + 48 = 0\,, \tag{9}$$

and of the two roots of this quadratic equation, only one is acceptable, namely

$$\kappa = \frac{77 - \sqrt{2281}}{38} \approx 0.77\,, \tag{10}$$

since the other, being greater than 2, would cause divergence in the integral $\int_0^x dyyG(y)$ (i.e. $\beta = -\kappa$ would be less than $-2$ for this unacceptable root).

The strong coupling for this solution is constant, since

$$\begin{aligned}\lambda_s(q^2) &\equiv \lambda F(q^2)G^2(q^2) = \lambda AB^2 \\ &= \left[\frac{3}{2(2-\kappa)} - \frac{1}{3-\kappa} + \frac{1}{4\kappa}\right]^{-1} \approx 0.91\,.\end{aligned} \tag{11}$$

## 4 Runge-Kutta System

Let us introduce the scaling

$$F(x) = a\overline{F}(x) \qquad G(x) = b\overline{G}(x)$$

obtaining

$$\overline{F}^{-1}(x) = \eta + \lambda ab^2 \left[ \frac{\overline{G}(x)}{x^2} \int_0^x dy \left( \frac{3y}{2} - \frac{y^2}{x} \right) \overline{G}(y) + \int_x^1 \frac{dy}{2y} \overline{G}^2(y) \right] \tag{12}$$

and

$$\overline{G}^{-1}(x) = \zeta - \frac{9}{4}\lambda ab^2 \left[ \frac{\overline{F}(x)}{x^2} \int_0^x dy y \overline{G}(y) + \int_x^1 \frac{dy}{y} \overline{F}(y)\overline{G}(y) \right], \tag{13}$$

where $\eta$ and $\zeta$ are constants. We are free to choose the scaling factors $a$ and $b$ such that

$$\lambda ab^2 = 1\,,$$

and there is still one scale free, which may be taken, for example, such that

$$\overline{F}(1) = 1\,. \tag{14}$$

Let us now drop the bars and rewrite the above equations in the form

$$F^{-1}(x) = \eta + \frac{3}{2}G(x)K(x) - G(x)L(x) + \frac{1}{2}\int_x^1 \frac{dy}{y} G^2(y) \tag{15}$$

and

$$G^{-1}(x) = \zeta - \frac{9}{4}F(x)K(x) - \frac{9}{4}\int_x^1 \frac{dy}{y} F(y)G(y)\,, \tag{16}$$

where

$$K(x) = \frac{1}{x^2}\int_0^x dy y G(y) \tag{17}$$

and

$$L(x) = \frac{1}{x^3}\int_0^x dy y^2 G(y)\,. \tag{18}$$

On differentiating the above four equations, we obtain

$$\begin{aligned}
\dot{F} &= F^2[-\tfrac{3}{2}\dot{G}K - \tfrac{3}{2}G\dot{K} + \dot{G}L + G\dot{L}] + \tfrac{1}{2}F^2G^2 \\
\dot{G} &= \tfrac{9}{4}G^2[\dot{F}K + F\dot{K}] - \tfrac{9}{4}FG^3 \\
\dot{K} &= G - 2K \\
\dot{L} &= G - 3L\,,
\end{aligned}$$

where $\dot{F} = \frac{dF}{dt} = x\frac{dF}{dx}$ etc., with $t = \log x$. After a little algebra, we can throw the first two of these equations into the form

$$\dot{F} = 3F(X-Y) - FZ(\tfrac{3}{2}X - Y) \qquad \dot{G} = ZG\,, \tag{19}$$

where

$$X = FGK \qquad Y = FGL \qquad Z = \frac{X(3X-3Y-2)}{\frac{4}{9} + X(\frac{3}{2}X - Y)}\,. \tag{20}$$

This system is suitable for an application of the Runge-Kutta method; but we must first address the question of the existence and multiplicity of the solutions, first of the differential system, and then of the integral equations (15)-(16) from which they were derived. We already know an (exact) solution, which, with the normalization Eq.(14), we may write

$$F(x) = x^{2\kappa} \qquad G(x) = g_0 x^{-\kappa}\,, \tag{21}$$

where $\kappa$ was defined in Eq.(10), and where (compare Eq.(8) )

$$g_0 = \frac{2}{3}\left(\frac{1}{\kappa} - \frac{1}{2-\kappa}\right)^{-\frac{1}{2}} \approx 0.96\,. \tag{22}$$

*A priori* we would expect there to be *four* free parameters for the differential system, corresponding say to the values of $F(1)$, $G(1)$, $K(1)$ and $L(1)$, from which the differential equations could step-by-step be integrated, for example by the Runge-Kutta method. In general, solutions of the differential system would not satisfy the requirements $x^2K(x) \to 0$ and $x^3L(x) \to 0$ as $x \to 0$. In fact the lower limits of the integrals in Eq.(17)-(18) would be incorrectly replaced by nonzero constants. Imposing the requisite boundary conditions at $x = 0$, we expect to reduce the number of arbitrary constants in the general solution from four to two. Since there is a scaling invariance that leaves $FG^2$ unchanged, this means that, after we have removed this trivial degree of freedom by imposing Eq.(14), we should still have one non-trivial free parameter. Where is it?

Let us construct power series for $F$ and $G$ about $x = 0$ of the form

$$\begin{aligned} F^{[n]}(x) &= x^{2\kappa}\sum_{j=0}^{n} f_j x^{j\rho} \\ G^{[n]}(x) &= x^{-\kappa}\sum_{j=0}^{n} g_j x^{j\rho}\,. \end{aligned} \tag{23}$$

We expect these series to have zero radius of convergence, so they have been truncated in the anticipation that they are asymptotic series — that is, for small values of $x$, there will be an optimal truncation point, $n$, for which the finite series is a good approximation. We will see shortly just how good the approximation can be.

As normalization condition we will take $f_0 = 1$ rather than Eq.(14) (which is no longer equivalent), and $g_0$ is determined by Eq.(22). Up to order $j = 1$ in the infrared series,

$$\begin{aligned} F^{[n]}(x) &= x^{2\kappa} + f_1 x^{2\kappa+\rho} + O(x^{2\kappa+2\rho}) \\ G^{[n]}(x) &= g_0 x^{-\kappa} + g_1 x^{-\kappa+\rho} + O(x^{-\kappa+2\rho})\,. \end{aligned} \tag{24}$$

The differential equations for $F$ and $G$ can now be used to determine the index $\rho$ and the ratio $g_1/f_1$. To see how this works in detail, define

$$\begin{aligned} P &= X(3X - 3Y - 2) \\ R &= \frac{4}{9} + X(\tfrac{3}{2}X - Y)\,, \end{aligned}$$

so that $Z = P/R$, see Eq.(20). Then Eq.(19) can be rewritten

$$\begin{aligned} \mathcal{A} &\equiv x^{-2\kappa}[xF'R - 3F(X-Y)R + F(\tfrac{3}{2}X - Y)P] = 0 \\ \mathcal{B} &\equiv x^{\kappa}[xG'R - GP] = 0\,. \end{aligned} \tag{25}$$

These equations, $\mathcal{A} = 0 = \mathcal{B}$, are rationalized, in the sense that, when the various functions are replaced by their asymptotic series, only products, and not quotients, of series occur. Moreover, the series involve integral powers of $y = x^\rho$ only. If we truncate the various series at $j = 1$, then the coefficients of the lowest powers in the expansions of $\mathcal{A}$ and $\mathcal{B}$ are homogeneous and linear in $f_1$ and $g_1$, and quartic in $\rho$. Of the four solutions for $\rho$, two are complex conjugate and they are of no further interest. Of the other two, one is $\rho = 0$, which is likewise uninteresting, but the fourth solution is

$$\rho = 1.9696386375 \qquad\qquad \frac{g_1}{f_1} = 0.7925939968$$

to 10 decimals. We may take $f_1$ as a new free parameter and successively determine the higher $f_j$ and $g_j$ in terms of it. More conveniently, we will choose $f_1 = 1$ and rewrite the IR asymptotic series as follows:

$$F^{[n]}(x) = x^{2\kappa}\left[1 + \sum_{j=1}^{n} f_j a^j x^{j\rho}\right]$$

$$G^{[n]}(x) = x^{-\kappa}\left[g_0 + \sum_{j=1}^{n} g_j a^j x^{j\rho}\right] , \tag{26}$$

where $a$ is an arbitrary constant, the parameter whose existence we adduced above.

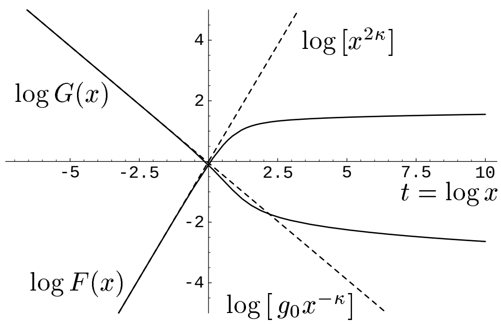


Figure 1: Gluon and Ghost Form Factors

How good is the asymptotic series? The coefficients $f_j$ and $g_j$ have been calculated in a Mathematica program, while the fact that they are proportional to $a^j$ can be proved by induction. Consider the case $a = -0.1$. In Fig. 1 we show log-log plots of $F(x)$ and $G(x)$. The abscissa is $t = \log x$ and the solid lines are the results of the Runge-Kutta routine. As the starting point, the IR series is evaluated with $n = 5$ at $x = 0.001$, which corresponds to $t = -6.9$. In the table below, the values of $F(0.01)$ and $G(0.01)$ are shown, first by evaluating the IR series with different values of the truncation point from $n = 0$ to $n = 10$.

IR asymptotic series:

| | n | F(0.01) | G(0.01) |
|---|---|---|---|
| | 0 | 0.0008357682484428076 | 33.04745145455120 |
| | 1 | 0.0008356721295869212 | 33.04429840664369 |
| | 2 | 0.0008356721250686792 | 33.04429783329862 |
| | 5 | 0.0008356721250696475 | 33.04429783325921 |
| | 10 | 0.0008356721250696475 | 33.04429783325921 |
| Runge-Kutta: | | 0.0008356721339488650 | 33.04429897448142 |

In the last line we give the result of using the the Runge-Kutta routine, run with 25 digit precision and 10 000 steps from $x = 0.001$ to $x = 0.01$. We notice that there is agreement to about 3 significant figures between the Runge-Kutta numbers and those derived directly from the asymptotic series, in the case $n = 0$, and to about 7 significant figures with $n$ from 2 and 10. This gives an idea of the accuracy involved and of the general reliability of the method.

## 5 Ultra-Violet Behaviour

Let us try the logarithmic asymptotic behaviours

$$F(x) \sim c \log^{\gamma} x \qquad G(x) \sim d \log^{\delta} x \,,$$

as $x \to \infty$. In Eq.(15)-(16) the last integrals alone give the leading order:

$$F^{-1}(x) \sim \frac{1}{2}\int_x^1 \frac{dy}{y} G^2(y) \qquad G^{-1}(x) \sim -\frac{9}{4}\int_x^1 \frac{dy}{y} F(y)G(y)\,,$$

from which we find

$$\frac{1}{c}\log^{-\gamma} x = -\frac{d^2}{2(1+2\delta)}\log^{1+2\delta} x$$
$$\frac{1}{d}\log^{-\delta} x = \frac{9cd}{4(1+\gamma+\delta)}\log^{1+\gamma+\delta} x\,,$$

and hence $1+\gamma+2\delta = 0$ and

$$cd^2 = -2(2\delta+1) = \tfrac{4}{9}(1+\gamma+\delta)\,.$$

This leads immediately to $\gamma = \frac{1}{8}$, $\delta = -\frac{9}{16}$ and $cd^2 = \frac{1}{4}$. Hence

$$F(x)G^2(x) \sim \tfrac{1}{4}\log^{-1} x\,. \tag{27}$$

Note that this asymptotic behaviour can only occur if $a \neq 0$, since for $a = 0$ IR power behaviour Eq.(21) is an exact solution that leads to a constant coupling (i.e. one that does not run). It is interesting that the asymptotic behaviour Eq.(27) simulates the expected asymptotic freedom, although of course the coefficient of the inverse logarithm is wrong, since gluon and quark loop corrections have been left out of account.

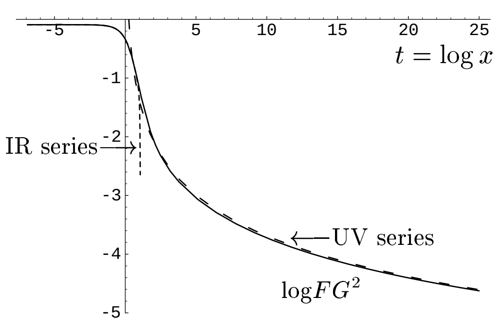


Figure 2: Running Coupling

In Fig. 2 we show the renormalization group invariant $\lambda\tilde{Z}_1^2 FG^2$, in the Landau gauge (in which $\tilde{Z}_1 = 1$), scaled to $\lambda = 1$, for the case $a = -0.1$, up to $x = \mathrm{e}^{25} \approx 7\times 10^{10}$. The fine dotted curve on the left is the IR asymptotic series, summed to $n = 5$, while the coarse dotted curve on the right is the UV asymptote Eq.(27).

## 6 Generalizations

Let us briefly indicate certain generalizations of the above scheme, some of which have already been implemented, some of which are in progress, and others of which appear to be problematical. In the first place, inclusion of a gluon loop in the gluon equation, with a bare three-gluon vertex, has indeed already been accomplished [6]. The infrared limit is not affected, but of course the coefficient of the logarithm in the ultraviolet is changed, as expected. So far so good, but the modified equations turn out to have lost some of the renormalization group invariance of the simpler equations, with the result that the running coupling depends on an extra parameter, besides $\Lambda_{\rm QCD}$. The deficiency is clearly the result of the crudity of the approximations; and Ball-Chiu expressions, in which the Slavnov-Taylor identities are taken into account, seem to be indicated. Indeed the original work of von Smekal et al.[5] incorporated just such Ball-Chiu Ansätze. A difficulty however is the occurrence of the spurious $\log x$ factor, Eq.(2). Von Smekal et al. avoided the difficulty by adopting differing prescriptions for the angular averaging in the infrared and in the ultraviolet parts of the integral. Although their recipe does suppress the symptom, in that the $\log x$ factor no longer occurs, it may reasonably be doubted that the disease itself has been cured, since the difficulty is present *even when the angular integrals are calculated exactly!*

Indeed, the next generalization is to perform the angular integrations without resorting to plain, or doctored angular averaging. The infrared behaviour can be calculated by hand, at least in the bare vertex approximation [7], and one finds the amusing result that the index $\kappa \to 1$. With the same method, it can be shown that there is no solution if one employs the von Smekal Ansatz for the ghost vertex instead of the corresponding bare vertex. Here is an unsolved problem: the bare vertex gives a sensible answer but the 'improved' vertex does not. Can a minor change in the Ansatz be found that restores harmony to the new singular ghost scenario, or must it be dismantled?

If the above disease can be cured properly, one still has the worry about the renormalization scale invariance of the equations when the three-gluon vertex is included. A Ball-Chiu Ansatz for this vertex alleviates but does not remove the pain (i.e. unphysical dependence on a supernumerary parameter). Evidently a more thoroughgoing Ansatz for the vertex is needed, perhaps one predicated on the need for renormalization scale invariance. Solution of the DS equations for other than limiting values in the infrared or ultraviolet involves the treatment of two-dimensional integral equations, i.e. equations involving two integrals, one over the radial and one over the angular variable. No reduction to differential equations is possible, and so the Runge-Kutta method is no longer applicable. The most promising of the available methods is projection of the unknown form factors onto a basis of known functions — whether these be Chebyshev polynomials or cubic splines, or something else — and the iteration of the system in the basis, i.e. as a nonlinear mapping on the space of coefficients of the polynomials. Work is in progress, in particular with Chebyshev polynomials, which were used with success in the pilot work involving simple angular averaging [6]. As it was in this pilot computation, Newton iteration will be employed to speed convergence.

The next step, which is an essential one before we can make meaningful comparisons with experiment, is the inclusion of quark loops. This seems a relatively easy hurdle, assuming the previous ones to have been successfully surmounted or sidestepped. However, similar allowance for the four-gluon vertex contributions seems fraught with difficulty of a new order of magnitude: all treatments to date are piously posited on the hope that these contributions, although they are indeed essential to exact gauge covariance, nevertheless have little numerical importance and may be neglected.

Having sketched a plan for improving the results obtained in this paper, it is perhaps not premature to ask if the notion of a soft gluon and a hard ghost makes physical sense or not. At least at first sight, this picture seems quite different from the received wisdom of a singular gluon and a negligible ghost. Indeed, it has been shown [8] that a gluon that vanishes in the infrared, *unaccompanied by a singular ghost,* is not appropiate for describing hadron phenomenology: neither chiral symmetry breaking nor quark confinement occurs in an acceptable manner. Moreover, it has been claimed [9] that a gluon propagator that vanishes like $p^2$ in the infrared implies the existence of a colour octet of 'particle-like' poles at zero mass in such $n$-point functions as the fully amputated, dressed three-gluon or quark-gluon vertices. Since such states have not been observed, it is asserted that a soft infrared gluon is empirically excluded. However, this analysis is weak, since it ignores the fact that a singular ghost can be expected to modify drastically any conclusion one might be inclined to draw from the mere observation that the gluon propagator vanishes in the infrared. In particular, the three-gluon vertex is modified, according to the Slavnov-Taylor identities, by a factor of the sort $G/F$, as compared with the bare form. Since this factor behaves like $p^{-6}$ in the infrared, we might expect the argument of Ref.[8] to be seriously affected. Indeed, we would not find 'particle-like' poles at all, but higher order singularities that would lead to a breakdown of the Lehmann representation and hence, perhaps, to technical confinement after all.

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